Reconstruction of Cubic Scenes from Images

Sheng Zhang

Shandong Provincial Institute of Land Surveying and Mapping, Jinan City, China
Correspondence: 59389530@qq.com

Abstract: This paper presents a new method for 3D reconstruction of indoor environments based on digital photograph. Firstly, it gets the exterior angle elements with collinear equation and the basal geometrical elements in the building. It chooses four points in the three mutual vertical lines: one is the point of intersection of the three lines and the other three are on the different lines. It gets exterior angle elements with the coordinates of the four points. Then it obtains the relative position of the photography center with one rectangle on the wall. So the paper can determine the position of photography center and the length of the other border of the rectangle. It chooses the rectangle of the other image based on the first rectangle to unify the scale of different images. Finally, it gets the coordinates of some especial points with the exterior elements of the images and restricting condition of man vision. It needs the one of the X, Y or Z of the special points and can get the other two coordinates with collinear equation. It evaluates the precision of the 3D coordinate through simulative and real data. Not only can it show the actual reconstruction result of the indoor scenes, it also can achieve the application extension of the method and reconstruct the model of the outdoor scenes.

Keywords: 3D reconstruction; exterior angle element, collinearity equation; geometric constraints

1. Introduction

With the rapid development of computer technology, graphics theory, photogrammetry technology, remote sensing technology and related technologies, the processing of 3D spatial data in two dimensions can no longer meet the needs of practical applications, and now users need to visualize, dynamically process, analyze, and display the various related geographic data. Urban 3D model is an important research direction of urban geographic information system. It is the constructing urban building model as one of the key issues that should be solved by urban 3D geographic information system.

Close-range photogrammetry is a branch of the photogrammetry and remote sensing discipline, which uses photography to determine the shape and motion state of the target [1]. The various objectives in the fields of industry, biomedicine, architecture and other fields of science are the subject of this discipline. Close-range photogrammetry can instantly acquire a large amount of physical and geometric information of the measured object; it is a non-contact measuring method that does not hurt the measurement target and can work in harsh environments; it is suitable for the determination of the shape and motion state of dynamic objects; it can provide measurement methods with high precision and reliability; it can offer various products based on three-dimensional space coordinates, including various data, graphics, images, digital surface models and three-dimensional dynamic sequence images [2]. Digital close-range photogrammetry technology has broad application prospects in the fields of cultural relics and archaeology, and engineering construction.

The digital close-range photogrammetry method has been used to carry out three-dimensional reconstruction of the external scene of the building [3]. This paper reconstructs the rule scene by digital close-range photogrammetry theory. The experiment proves that the proposed method is feasible.

2. Technical Scheme

(1) Extraction of feature points on three mutually perpendicular lines.

(2) Acquisition of orientation angle elements of the image. Also the mathematical equations are solved which include the image orientation angle elements and the feature point image coordinates; the correctness and accuracy of the obtained data are tested [4].

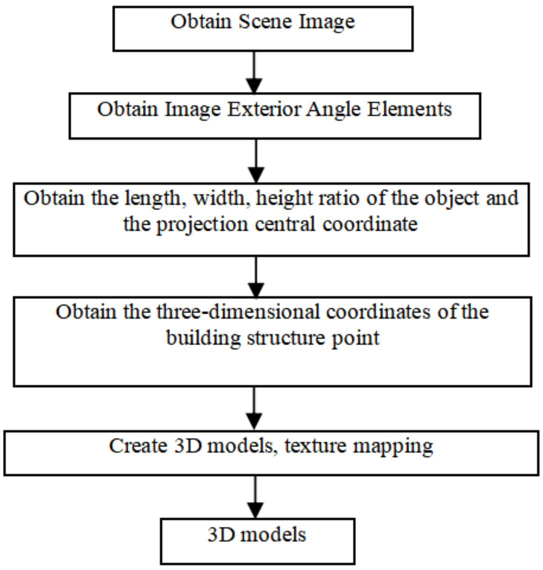

Figure 1. Technical Process

(3) Obtaining of the length, width, height, and the projection central coordinate. This process carries out feature points extraction that can reflect the length, width, and height of the building and also establishes mathematical models that include the length (width), elevation, and image orientation elements of the building, and solves the equations to obtain the length, width, and height of the building, proportion and corresponding coordinates of the photography center, verifies the accuracy of the building's length, width, height and relative coordinates of the photography center.

(4) Using the coplanar constraints and the values of the exterior orientation elements that have been obtained, the collinearity equation and constraints are used to obtain the three-dimensional coordinates of the feature points of the detailed structure of the building [5].

(5) Construction of the three-dimensional model. In this process, a three-dimensional geometric model is established through obtaining the three-dimensional coordinates of the structural points and the coordinates of the main feature points in the custom coordinate system.

3. Acquisition of Image Orientation Angle Elements

It is known that the image has three internal orientation elements x0, y0, and f, and the three angle elements of the image can be calculated by the obtained pixel coordinates of Pc, P1, P2, and P3. The relationship between the observed value and the unknown number in the collinear equation is a nonlinear function. In order to facilitate the calculation, the nonlinear function expression needs to be expanded into a linear form by Taylor's formula. The specific solution is as follows, where ai, bi, ci (i=1, 2, 3) are the 9-direction cosine composed of the 3 exterior orientation angle elements of the image [6].

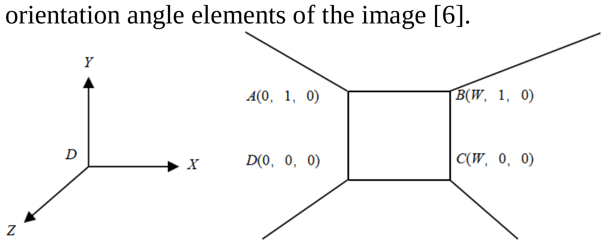

Figure 2. Indoor structure diagram

According to the collinear equation:

$$(x_0 - x_c)(y_1 - y_c) - (y_0 - y_c)(x_1 - x_c) - f \bullet (y_1 - y_c) \bullet a_1 / a_3 + f \bullet (x_1 - x_c) \bullet a_2 / a_3 = 0 \quad (1)$$

Let:

$$A = (x_0 - x_c)(y_1 - y_c) - (y_0 - y_c)(x_1 - x_c) \quad (2)$$

So:

$$A \bullet a_3 - f \bullet (y_1 - y_c) \bullet a_1 + f \bullet (x_1 - x_c) \bullet a_2 = 0 \quad (3)$$

Let:

$$F(\varphi, \omega, \kappa) = A \bullet a_3 - f \bullet (y_1 - y_c) \bullet a_1 + f \bullet (x_1 - x_c) \bullet a_2 \quad (4)$$

$$V_F - F(\varphi_0, \omega_0, \kappa_0) = a'_1 \bullet \Delta\varphi + b'_1 \bullet \Delta\omega + c'_1 \bullet \Delta\kappa \quad (5)$$

Similarly, according to equations (5) and (6), let:

$$B = (x_0 - x_c)(y_2 - y_c) - (y_0 - y_c)(x_2 - x_c) \quad (6)$$

$$M(\varphi, \omega, \kappa) = B \bullet b_3 - f \bullet (y_2 - y_c) \bullet b_1 + f \bullet (x_2 - x_c) \bullet b_2 \quad (7)$$

$$V_M - M(\varphi_0, \omega_0, \kappa_0) = a'_2 \bullet \Delta\varphi + b'_2 \bullet \Delta\omega + c'_2 \bullet \Delta\kappa \quad (8)$$

$$C = (x_0 - x_c)(y_3 - y_c) - (y_0 - y_c)(x_3 - x_c) \quad (9)$$

$$N(\varphi, \omega, \kappa) = C \bullet c_3 - f \bullet (y_3 - y_c) \bullet c_1 + f (x_3 - x_c) \bullet c_2 \quad (10)$$

$$V_N - N(\varphi_0, \omega_0, \kappa_0) = a'_3 \bullet \Delta\varphi + b'_3 \bullet \Delta\omega + c'_3 \bullet \Delta\kappa \quad (11)$$

The matrix form of the error equation is:

$V = AX - L$

Expression of the solution of the normal equation:

$X = (A^T A)^{-1} A^T L$

Starting with the initial values of φ, ω, and κ, and the sum of the approximation of the unknown and the correction of the previous iteration is used as the new approximation (formula 1) for each iteration, and the calculation process is repeated. The new correction number is repeated until the correction is less than the required precision value. Whether the final result converges or not depends on the initial values of φ, ω, and κ.

4. Obtaining Thelength, Width, Height and The Photographic Central Coordinate

4.1. Mathematical Model

Based on the collinear equation, the ratio of height and length (width) of the house can be obtained by coplanar constraints and the four vertices of the rectangle (Fig. 1), and at the same time, the relative position of the photographic center can be obtained [7].

If the three inner orientation elements x0, y0, and f are known, take x0=0, y0=0, and find the three angle elements of the image φ, ω, and κ. Let AD = H = 1, AB = W, from A(0, 1, 0), B(W,1,0), C(W,0,0), D(0, 0, 0) and the collinear equation, the values of the coordinates XS, YS, ZS and W of the photographing center S can be obtained. See Mathematical Formulas from (12) to (15).

$$\begin{aligned} x_A - 0 &= -f \frac{a_1(0 - X_S) + b_1(1 - Y_S) + c_1(0 - Z_S)}{a_3(0 - X_S) + b_3(1 - Y_S) + c_3(0 - Z_S)} \\ y_A - 0 &= -f \frac{a_2(0 - X_S) + b_2(1 - Y_S) + c_2(0 - Z_S)}{a_3(0 - X_S) + b_3(1 - Y_S) + c_3(0 - Z_S)} \end{aligned} \quad (12)$$

$$\begin{aligned} x_B - 0 &= -f \frac{a_1(W - X_S) + b_1(1 - Y_S) + c_1(0 - Z_S)}{a_3(W - X_S) + b_3(1 - Y_S) + c_3(0 - Z_S)} \\ y_B - 0 &= -f \frac{a_2(W - X_S) + b_2(1 - Y_S) + c_2(0 - Z_S)}{a_3(W - X_S) + b_3(1 - Y_S) + c_3(0 - Z_S)} \end{aligned} \quad (13)$$

$$\begin{aligned} x_C - 0 &= -f \frac{a_1(W - X_S) + b_1(0 - Y_S) + c_1(0 - Z_S)}{a_3(W - X_S) + b_3(0 - Y_S) + c_3(0 - Z_S)} \\ y_C - 0 &= -f \frac{a_2(W - X_S) + b_2(0 - Y_S) + c_2(0 - Z_S)}{a_3(W - X_S) + b_3(0 - Y_S) + c_3(0 - Z_S)} \end{aligned} \quad (14)$$

$$\begin{aligned} x_D - 0 &= -f \frac{a_1(0 - X_S) + b_1(0 - Y_S) + c_1(0 - Z_S)}{a_3(0 - X_S) + b_3(0 - Y_S) + c_3(0 - Z_S)} \\ y_D - 0 &= -f \frac{a_2(0 - X_S) + b_2(0 - Y_S) + c_2(0 - Z_S)}{a_3(0 - X_S) + b_3(0 - Y_S) + c_3(0 - Z_S)} \end{aligned} \quad (15)$$

It is known that the four points A, B, C, and D under the object space coordinate system (custom) can obtain eight equations according to the collinear equation, among which there are four unknowns XS, YS, ZS, W, using the formula (12), (13), (14), (15) can establish an adjustment model to calculate the coordinates of the photography center and W [8].

Similarly, for another image containing information on the length L of the house and the height H (L, H is a rectangle of two adjacent sides), the new model center coordinates and the value of L can be calculated using the same model.

4.2. Model Solution

For equations (12), (13), (14), (15), eight equations, four unknowns, can be solved using the principle of least squares adjustment. When x_A, y_A, x_B, y_B, x_C, y_C, x_D, y_D, f, a_i, b_i, c_i (i = 1, 2, 3) are known, XS, YS, ZS, and W will be obtained. Where φ, ω, and κ are the exterior orientation angle elements of the image, and a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, and c_3 are elements of the matrix R.

According to formula 12-15:

$$x_A a_3 X_S / f - x_A b_3 / f + x_A b_3 Y_S / f + x_A c_3 Z_S / f + a_1 X_S + b_1 Y_S + c_1 Z_S - b_1 = 0$$

$$(y_A a_3 / f + a_2) \bullet X_S + (y_A b_3 / f + b_2) \bullet Y_S + (y_A c_3 / f + c_2) \bullet Z_S + 0 \bullet W = y_A b_3 / f + b_2$$

$$(x_B a_3 / f + a_1) \bullet X_S + (x_B b_3 / f + b_1) \bullet Y_S + (x_B c_3 / f + c_1) \bullet Z_S + (-x_B a_3 / f - a_1) \bullet W = x_B b_3 / f + b_1$$

$$(y_B a_3 / f + a_2) \bullet X_S + (y_B b_3 / f + b_2) \bullet Y_S + (y_B c_3 / f + c_2) \bullet Z_S + (-y_B a_3 / f - a_2) \bullet W = y_B b_3 / f + b_2$$

$$(x_C a_3 / f + a_1) \bullet X_S + (x_C b_3 / f + b_1) \bullet Y_S + (x_C c_3 / f + c_1) \bullet Z_S + (-x_C a_3 / f - a_1) \bullet W = 0$$

$$(y_C a_3 / f + a_2) \bullet X_S + (y_C b_3 / f + b_2) \bullet Y_S + (y_C c_3 / f + c_2) \bullet Z_S + (-y_C a_3 / f - a_2) \bullet W = 0$$

$$(x_D a_3 / f + a_1) \bullet X_S + (x_D b_3 / f + b_1) \bullet Y_S + (x_D c_3 / f + c_1) \bullet Z_S + 0 \bullet W = 0$$

$$(y_D a_3 / f + a_2) \bullet X_S + (y_D b_3 / f + b_2) \bullet Y_S + (y_D c_3 / f + c_2) \bullet Z_S + 0 \bullet W = 0$$

List the equation according to the principle of least squares adjustment：

$$A^{\mathrm{T}} PAX = A^{\mathrm{T}} PL$$

Where: P is the weight matrix of the observed values, and P is considered to be the identity matrix, thus obtaining the expression of the solution of the normal equation [9]:

$$X = (A^{\mathrm{T}} A)^{-1} A^{\mathrm{T}} L$$

Among them:

$$X = \begin{bmatrix} X_S \\ Y_S \\ Z_S \\ W \end{bmatrix}$$

$$A = \begin{bmatrix} l_1 & m_1 & n_1 & p_1 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ l_8 & m_8 & n_8 & p_8 \end{bmatrix} \quad L = \begin{bmatrix} F_1 \\ \vdots \\ \vdots \\ F_8 \end{bmatrix}$$

5. Experimengtal Result

In this paper, six points on the three vertical lines (under the object space) in the image are selected, and the angle elements of the image are calculated by their image coordinates. When select the feature points on each line in the image [10], you'd better select two points with the largest possible distance. The solution error of the feature points with smaller distance is smaller than the feature points with larger distance. As shown in FIG. 3, the six feature points from p11 to p32 are selected when calculate the angles elements of the image, and the rectangle is selected when calculate the coordinates of the center of the photograph.

Figure 3. Selection of feature points and the rectangle and rectangles

Table 1 shows the theoretical and calculated values of the actual cuboid model. The coordinates of the photographing center are: (12.876, 15.510, 12.998), and the unit is cm. Figure 3 shows the results of the reconstruction of the cuboid model. The reconstruction method used in this paper can't obtain the absolute model.

Table 1. Result of the cuboid model experiment

	Length (cm)	Width (cm)	Height (cm)
Theoretical Value	8.51	7.50	7.50
Calculated value	8.546	7.447	

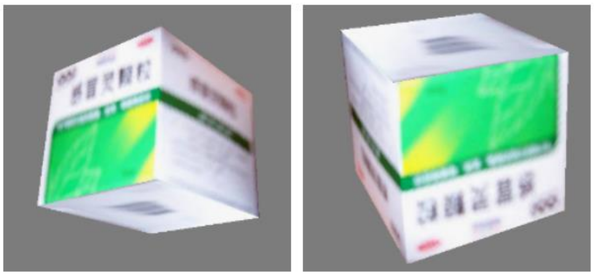

Figure 3. Reconstruction results of rectangular model

6. Conclusion

Using the images acquired by ordinary digital cameras, based on the collinear equation, the building scenes were reconstructed, and a three-dimensional model of the scene with real scale was obtained, which opened up a new way to reconstruct the building scenes with digital images. After experimenting with simulated data and actual data, the following conclusions were obtained:

(1) It is entirely feasible to use the three straight lines in the image (the objects are perpendicular to each other in the object space) to obtain the orientation angle elements of the image. The orientation angle elements of the image can be obtained by four feature points on three mutually perpendicular lines (under the object space) having a common intersection (one of which is their common intersection and the other three are selected on each line). It is also possible to obtain the orientation angle elements of the image by any of six feature points on the line perpendicular to each other (under the object space) (two points are selected on each line).

(2) In indoor (outdoor) scene reconstruction, it is a feasible and convenient way to obtain the relative coordinates of the image projection center by the rectangle under the object space, that is, to obtain the exterior line elements of the image. Each image is selected with a rectangle to obtain the exterior line elements of the image. The ratio between the lengths of the rectangles in the two images can be used to determine the scale factor between different 3D models, passing through a vertex of the two rectangles. The positional relationship between the vertices in the lower left corner of this paper is used to obtain the conversion relationship between different coordinate systems, thus solving the connection problem In order to view the results of the experiment, the height of the cuboid model is regarded as the known.
between adjacent models.

(3) This paper manually gives the image coordinates of the structure points and the constraints (coplanar constraints) information to obtain the three-dimensional coordinates of the indoor structure points. The complexity of the indoor scene structure and the need for reconstruction determine the complexity of reconstructing the indoor structure. Factors affecting the accuracy of the three-dimensional coordinates of the structure points are as follows: the constraints given, the image coordinates of the points, and the exterior orientation elements of the image.

(4) Using the method proposed in this paper, the indoor and outdoor scenes of the actual building are reconstructed, and a realistic and textured three-dimensional model is obtained. Therefore, it is feasible to use the indoor scene reconstruction method proposed in this paper.

References

[1] El-Hakim S F, Brenner C, Roth G. A multi-sensor approach to creating accurate virtual environments. *ISPRS journal of photogrammetry and remote sensing*, **1998**, 53(6): 379-391.
[2] Yu Y, Ferencz A, Malik J. Extracting objects from range and radiance images. *IEEE Transactions on Visualization and Computer Graphics*, **2001**, 7(4): 351-364.
[3] Sequeira V, Gonçalves J G M, Ribeiro M I. 3D reconstruction of indoor environments. Proceedings of 3rd IEEE International Conference on Image Processing. IEEE, **1996**, 2: 405-408.
[4] Wang R, Luebke D. Efficient reconstruction of indoor scenes with color. Fourth International Conference on 3-D Digital Imaging and Modeling, 3DIM 2003. Proceedings. IEEE, **2003**: 402-409.
[5] Whitaker R T, Gregor J, Chen P F. Indoor scene reconstruction from sets of noisy range images. Second International Conference on 3-D Digital Imaging and Modeling (Cat. No. PR00062). IEEE, **1999**: 348-357.
[6] Chang W C, Lee S A. Real-time feature-based 3D map reconstruction for stereo visual guidance and control of mobile robots in indoor environments. 2004 IEEE International Conference on Systems, Man and Cybernetics (IEEE Cat. No. 04CH37583). IEEE, **2004**, 6: 5386-5391.
[7] Hoshino J, Uemura T, Masuda I. Region-based reconstruction of an indoor scene using an integration of active and passive sensing techniques. Proceedings Third International Conference on Computer Vision. IEEE, **1990**: 568-572.
[8] Caprile B, Torre V. Using vanishing points for camera calibration. *International journal of computer vision*, **1990**, 4(2): 127-139.
[9] Liebowitz D, Zisserman A. Combining scene and auto-calibration constraints. Proceedings of the Seventh IEEE International Conference on Computer Vision. IEEE, **1999**, 1: 293-300.
[10] Sturm P. A method for 3d reconstruction of piecewise planar objects from single panoramic images. Proceedings IEEE Workshop on Omnidirectional Vision (Cat. No. PR00704). IEEE, **2000**: 119-126.